Quantum Mechanics as a Classical Theory V: The Quantum Schwartzchild Problem

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Abstract

In this continuation paper, we apply the general relativistic quantum theory for one particle systems, derived in paper II of this series, to a simple problem: the quantum Schwartzchild problem, where one particle of mass m gravitates around a massive body. The results thus obtained reveal that, in the realm of such a theory, the negative mass conjecture we made in paper IV of this series is, indeed, adequate. It is shown that gravitation is responsible for the loss of energy quantization. We relate this property with the ideas of irreversibility and time arrow.

1 Introduction

In the previous paper (hereafter IV), we conjectured that it is possible to describe Nature appealing to negative mass particles (antiparticles). All the experiments the "orthodox" interpretation might explain can also be explained by our interpretation. The experiment where particle and antiparticle travel under the influence of a homogeneous magnetic field was one we explained in details. We saw that the electromagnetic interaction alone is not capable of deciding about mass sign, since it is concerned only with the ratio e/m and the velocity. We also argued that a theory that accounts for gravitational effects will be capable of such a decision.

In paper II of this series[1, 2, 3], we developed a general relativistic quantum theory for ensembles composed of one particle systems. This theory can be considered an immediate generalization of Klein-Gordon's and Dirac's special relativistic theories. It includes Einstein's equations as part of the system of equations one shall solve and, thus, takes into account gravitation. The application of this theory to a problem where only gravitational effects are present will decide unambiguously about the correctness of the conjecture made in IV.

In the present paper we will apply the above mentioned theory to what we call the quantum Schwartzchild problem. The classical version of this problem is well known: a massive body leads to a geometrical distortion in space-time structure that is felt by a test particle with vanishing mass.

The quantum counterpart of this problem is as follows: we suppose that the initial conditions related with the test particle might not be known. It is thus necessary to approach the problem statistically. The resulting statistical description shall account for the test particle probability distribution over space-time. The function that emerges from the calculations shall represent the probability amplitude related with the test particle being somewhere in tridimensional space at some instant of time - an event probability amplitude.

As with the electron clouds of the hydrogen atom problem, the test particle becomes represented by a continuous (probability) density distribution. This implies that all its properties, such as the mass or the charge, shall be also considered as continuously distributed in space-time.

In the second section, we state the problem mathematically and solve it exactly.

The third section will be concerned with the interpretations of the results obtained in the second section.

We then make our conclusions. We discuss the effect of gravity upon quantization and it is shown that gravitation is related with the extinction of quantization. This property is paralleled with the idea of irreversibility and time arrow. The superposition principle is also discussed.

In the appendix, some of the more restringent classical arguments against[5, 6] "antigravity" are discussed in details and it is shown that they are not adequate.

2 The Problem

We showed, in paper II of this series, that the system of equations we shall solve when considering a general relativistic quantum problem is given by

$$\frac{-\hbar^2}{2mR}\Box R + V - \frac{mc^2}{2} + \frac{\nabla_\beta S \nabla^\beta S}{2m} = 0; \tag{1}$$

$$G_{\mu\nu} = -\frac{8\pi G}{c^2}\left[T_{(M)\mu\nu} + T_{(Q)\mu\nu}\right], \tag{2}$$

where the functions R and S are related to the probability amplitude by

$$\psi\left(x\right) = R\left(x\right)\exp\left(iS\left(x\right)/\hbar\right), \tag{3}$$

G is Newton's gravitational constant, $T_{(M)\mu\nu}$ is the matter energy-momentum tensor, $T_{(Q)\mu\nu}$ is the energy-momentum tensor of the statistical field[1, 2, 3]

given, in terms of R and S as

$$T_{(Q)\mu\nu} = mR\,(x)^2\,\frac{\nabla_\mu S}{m}\frac{\nabla_\nu S}{m} \tag{4}$$

and $G_{\mu\nu}$ is Einstein's tensor (here m represents the modulus of the mass appearing in the Klein-Gordon's equation).

In the problem in which we are interested there is only the gravitational force. Then, we expect only the statistical field's energy-momentum tensor to appear in the right side of equation (2). Our specific problem demands that we rewrite equations (1) and (2) as

$$\frac{-\hbar^2}{2mR}\Box R - \frac{mc^2}{2} + \frac{\nabla_\beta S \nabla^\beta S}{2m} = 0 \tag{5}$$

and

$$G_{\mu\nu} = -\frac{8\pi G}{c^2}\rho\,(r,\tau)\,u_\mu u_\nu, \tag{6}$$

where we used the following conventions:

$$\rho\,(x) = mR\,(x)^2 \;\; ; \; u_\mu = \frac{\nabla_\mu S}{m}. \tag{7}$$

It is preferable to treat this problem using comoving coordinates defined by the line element

$$ds^2 = c^2 d\tau^2 - e^{w(r,\tau)}dr^2 - e^{v(r,\tau)}\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{8}$$

where τ is the particle proper time, (r,θ,ϕ) its spherical-polar coordinates and $w\,(r,\tau)$, $v\,(r,\tau)$ the functions we shall obtain to fix the metric. Looking at equation (5) we can see that, in the comoving coordinate system, we shall have

$$\nabla_\mu S \nabla^\mu S = m^2 c^2 \Rightarrow u_\mu u^\mu = c^2, \tag{9}$$

implying that

$$S\,(x) = \pm mc^2 \tau. \tag{10}$$

As the coordinate system is comoving, we might put $u^\mu = (u^0, 0, 0, 0)$ and the statistical field's energy-momentum tensor becomes

$$T_{(Q)00} = \rho\,(r,\tau)\,c^2 \; ; \; T_{(Q)\mu\nu} = 0 \text{ if } \mu \neq 0 \text{ or } \nu \neq 0. \tag{11}$$

Einstein's equations can now be written explicitly as

$$-e^{-w}\left(v'' + \frac{3}{4}v'^2 - \frac{1}{2}w'v'\right) + e^{-v} + \frac{1}{4}\dot{v}^2 + \frac{1}{2}\dot{v}\dot{w} = 8\pi G\rho; \tag{12}$$

$$v' + \frac{1}{2}w' - \frac{1}{2}\dot{w}\,v' = 0; \tag{13}$$

$$e^{w}\left(\ddot{v}+\frac{3}{4}\dot{v}^{2}+e^{-v}\right)-\frac{1}{4}v'^{2}=0; \tag{14}$$

$$e^{v}\left(\ddot{v}+\frac{1}{4}\dot{v}^{2}+\frac{1}{4}\dot{v}\dot{w}+\frac{1}{2}\ddot{w}+\frac{1}{4}\dot{w}^{2}\right)+e^{v-w}\left(\frac{1}{4}w'v'-\frac{1}{2}v''-\frac{1}{2}v'^{2}\right)=0, \tag{15}$$

where the line and the dot indicate derivatives regarding variables r and τ, respectively. We can solve the last three equations if we put[4]

$$e^{w}=\frac{e^{v}v'^{2}}{4}\ ;\ e^{v}=\left[F\left(r\right)\tau+G\left(r\right)\right], \tag{16}$$

where $F(r)$ and $G(r)$ are arbitrary functions of r. From equation (12) we get the density function $\rho\left(r,t\right)$ with its explicit dependence on the metric given by the functions $F(r)$ and $G(r)$:

$$\rho\left(r,\tau\right)=\left[\frac{1}{6\pi G}\right]\frac{F\left(r\right)F'\left(r\right)}{\left[F\left(r\right)\tau+G\left(r\right)\right]\left[F'\left(r\right)\tau+G'\left(r\right)\right]}. \tag{17}$$

To solve our primary system of equations (1-2) we still have to solve equation (1) that becomes, using (9) and (10),

$$\Box R\left(r,\tau\right)=0. \tag{18}$$

We shall stress at this point that equation (18) is highly non-linear. The functions that define the density also define the metric. These functions will equally well be present in the D'Alambertian operator. Moreover, the function $R(r,\tau)$ is the square-root of the density function given by (17).

We can solve this equation using the degree of freedom we have in the choice of the arbitrary function $G(r)$; choosing it to be identically zero

$$G\left(r\right)=0, \tag{19}$$

we obtain the result

$$R\left(r,\tau\right)=N\sqrt{\frac{1}{6\pi mG}}\frac{1}{\tau}, \tag{20}$$

where N is a normalization constant.

Replacing these results in the expression (8) for the metric, we get

$$ds^{2}=c^{2}d\tau^{2}-\left(\frac{4}{9}\frac{F'\left(r\right)^{2}\tau^{2}}{F\left(r\right)^{2/3}\tau^{2/3}}\right)dr^{2}-\left[F\left(r\right)\tau\right]^{4/3}\left(d\theta^{2}+\sin^{2}\theta d\phi^{2}\right), \tag{21}$$

that can be further reduced to the format

$$ds^{2}=c^{2}d\tau^{2}-\tau^{4/3}\left[d\chi^{2}-\chi^{2}\left(d\theta^{2}+\sin^{2}\theta d\phi^{2}\right)\right], \tag{22}$$

where

$$\chi\left(r\right)=F\left(r\right)^{2/3}. \tag{23}$$

Collecting all the above results, the probability amplitude for the quantum Schwartzchild problem becomes

$$\psi_P(\tau) = N\sqrt{\frac{1}{6\pi mG}}\frac{e^{-imc^2\tau/\hbar}}{\tau}\ ;\ \psi_A(\tau) = N\sqrt{\frac{1}{6\pi mG}}\frac{e^{+imc^2\tau/\hbar}}{\tau} \tag{24}$$

in the comoving coordinate system, representing particle and antiparticle solutions (here we are supposing the massive body to be made of positive mass but this is not crucial; the important thing here is the difference in the signs of the probability amplitudes phases).

The interpretation of equation (24) is unambiguous. The particle solution represents the probability density that a particle, in its rest frame, is traveling in the direction of the massive body along its own world-line. The solution represented by ψ_A gives a particle that is traveling along its world-line, but in the contrary proper time direction (figure 1a). We might draw a better picture of what is happening if we take a look at the projection of these world trajectories into three dimensional space. It becomes clear that, while the particle is falling freely over the massive body, the antiparticle moves farther in what we might call free ejection.

To adequate this to a description where proper time flows only in the positive direction, it is necessary that we invert the time coordinate of the plus sign solution. This will make the trajectory of the plus sign solution coincide with the other particle trajectory. To achieve again the free ejection in three dimensional space we shall also invert the sign of the mass (figure 1b). This is precisely the same procedure already done in the appendix of the previous paper.

We stressed, in the last paper, that these interpretations are mathematically, though not physically, equivalent. This theory thus presents a symmetry; it says that a positive mass particle flowing backward in time is equivalent to a negative mass antiparticle flowing onward. It then turns out that particles are attracted by the gravitational field of a positive mass body, while antiparticles are repelled. This fixes the signs of masses.

The resulting metric is of a Robertson-Walker type

$$ds^2 = c^2 d\tau^2 - R(\tau)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right], \tag{25}$$

with $k = 0$; meaning that three space, with the radius defined by (23), is flat. The Hubble constant is easily computed and gives the usual value

$$H = \frac{1}{\tau}, \tag{26}$$

as expected for this problem.

3 Conclusions

We might derive many interesting consequences from the previous formalism and the example above.

The first thing we note when looking at system (1-2) is that this system is highly non-linear. This feature can be exemplified by expression (18) of the quantum Schwartzchild problem. We do not have, in general, a linear eigenvalue equation.

Energy quantization is, however, strictly related with the quantum equations being linear eigenvalue ones. If a system is to be described by the system of equations (1-2), then we shall not expect energy quantization to take place. Although some systems might still keep, in some very restricted range, their property of quantization, this shall be the exception rather than the rule.

The tendency of the energy spectrum to lose its discreteness character might be associated with a departure from the equilibrium by the system. Indeed, in our previous calculations[1, 2, 3], we started with the classical Liouville's equation and then obtained the quantum equation for the probability density. This density was then written as

$$\rho\left(x,x'\right)=\psi^{\dagger}\left(x'\right)\psi\left(x\right), \tag{27}$$

and we were able to derive Schrödinger's equation (non-relativistic case) for the probability amplitude. When we assume a stationary, pure state, configuration for the system

$$\psi\left(\mathbf{x},t\right)=\varphi\left(\mathbf{r}\right)e^{-iEt/\hbar}, \tag{28}$$

we are also assuming that the probability density does not depend explicitly on the time. This is equivalent to assume that the system is in one of its equilibrium states.

With the generalization introduced by system (1-2), it is, in general, not possible to admit a time dependence like (28) above. The superposition principle shall not be valid since it is based in the linear character of the equations. The gravitational field, thus, plays the role of removing the system from equilibrium.

This might be seen in the example of the quantum Schwartzchild problem. If we are to solve this problem, non relativistically, in flat space, we shall use Schrödinger's equation with the gravitational potential

$$V=-\frac{GM}{r}. \tag{29}$$

We then get a level scheme, basically a hydrogen-like one, where the levels will be very nearly spaced, but we still get quantization. When the problem is solved, using system (1-2), all the quantization disappears and we are left with a collapse-ejection-like solution. We stress that this feature is expected for almost all the problems (note that we also do not have solutions depending on the angles). We might thus say that we are lucky in living in a world where

gravitation does not play a predominant role. Our world might be thought as a world in equilibrium only to a good approximation, but not strictly.

Gravity pushes everything to non-equilibrium states until, probably, collapse-ejection takes place. In strong gravitational fields we shall not expect to meet atoms as we face them in our everyday life. They shall not be in their stable configurations.

When driving all systems to non-equilibrium configurations, gravitation introduces a time arrow . In the example of the atom cited above, even if the atom (hydrogen, for simplicity) is distant from any other massive body, it is suficient that the proton and the electron have finite masses for introducing into the system a finite mean lifetime. Of course, since their gravitational field is very feeble, the system's mean lifetime will be enormous. We might also consider that electromagnetic forces are also present, implying that the solution obtained in the last section will not strictly apply. However, we do not expect the qualitative analysis to be much different. We might, of course, find in Nature other forces capable of stabilizing a system. These forces, however, will compete with gravitation.

We thus began trying to consolidate the negative mass conjecture and arrived at a striking different world. We might see that the world suggested by these results is very distinct from the one we are familiarized when we consider the questions about the cosmological models of our Universe and compare their traditional answers with the ones emerging from our picture of Nature. This will be left for another work.

It is important to stress here that, for the epistemological framework adopted by this series of papers, the words (and worlds) *classical* and *quantum* are not oposed to each other; they are, on the contrary, complemetary views of the same (classical) Nature. Quantum Mechanics is here merely a name for a classical statistical mechanics (from the ontological point of view) performed in configuration space. Indeed, for the orthodox quantum mechanical view, the notion of geodesic is not admissible[6]. This theory, therefore, does not suffer from that sort of "incompatibilities" emphasized by a great number of authors[7]- [11]. That is why it was possible to join them into just one theory without modifying their structures[12, 13].

One last word is, nevertheless, appropriate. This theory might explain why our universe seems to have many more particles than antiparticles. The property of gravitation to be an attractive (long range) force between similar entities while being a repulsive force between different ones implies that the Universe tends to split into two distinct parts[14]. Considering that some radiation era existed, when pairs of particle and antiparticle were being created, their mutual repulsion might have strongly impeled them to occupy distinct portions of the Universe in a cumulative process[15].

It is extremely interesting to see how two highly different worlds emerged from two distinct interpretations of the special relativistic formalism of quantum mechanics.

There have appeared in the literature since 1957 many arguments against the notion of "antigravity"[5, 6, 16, 17]. Many of them are based in gedanken experiments and, as we have shown in the appendix, cannot be sustained. Some experiments are now in progress to measure the gravitational acceleration of antiparticles[6]. They will be of utmost importance to prove, or disprove, the negative mass conjecture.

A Antigravity

The first ideas of "antigravity" came into play in 1957 with the pioneering work of Bondi[18] who asked if general relativity could accomodate the notion of negative mass. Yet, in the following year, the first argument against negative masses appeared with Morrison's celebrated paper[5, 19]. Other arguments followed in the subsequent years[16, 17]. Morrison's argument, however, might be considered the most restricting one, since it is related with the notion of energy conservation and is the one we will consider in more detail in this appendix.

Morrison's argument is based on a *gedanken* experiment. Thus, before analyzing the argument itself, it is interesting to say some words about the *status* we shall ascribe to such an approach.

Gedanken experiments are not, as one may think, a peculiarity of our century. Indeed, it comes from the aristotelian idea that one might scrutinize Nature by means of thought alone and is based on the assumed homogeneity of human's reasoning and Nature's order[20]. This idea, however, does not take into account that, when performing any *gedanken* experiment, we are also approaching Nature by means of some theory (or proto-theory). This is nothing but the dispute between Hume[21] and Kant[22], to say but a few. This can be verified by historical examples.

The first one we cite is the debate between aristotelians and galileans about the free fall of bodies. Both schools based their arguments on the same mental experiment[20], the projectile falling from the mast of a ship in movement. Their conclusions were, however, opposite. The aristotelians, based on their conceptions of Nature, should not accept that the *impetus* (movement, in modern terms) of the ship was transferred to the projectile, and so, once left it will never fall at the base of the mast. The galileans, however, had a picture of Nature that could accomodate the *impetus* transference; for them the projectil will ever fall at the base of the mast. Although we now know the galileans were correct, none of them has ever done the experiment[20] at that time. It was a question of principle.

The other famous series of *gedanken* experiments are those related with the dispute between Einstein and Bohr. One of them, the EPR[23], playing a relevant role on the epistemological development of quantum mechanics. Looking at them carefully might convince one that both sides are full of epistemological prejudices[24, 25].

The considerations made above do not intend to deny the importance of *gedanken* experiments (some times they are the only ones left). They have just the intention of clarifying that these experiments shall not be used to exclude Nature behavior but only exclude Nature behavior *with respect to some theory or approximation.*

The arguments above might be considered a runaway solution. To avoid this misinterpretation we will present Morrison's arguments and show that they can be used also to get the opposite answer.

A.1 Morrison's Argument:

Morrison bases his arguments about negative mass beginning with an experiment with positive masses and then extending it to embrace negative ones. He then shows that this leads to a violation of the energy conservation principle. We begin, thus, with this first experiment where only positive masses are considered.

The experimental setup is shown in figure 2. It consists of a well-balanced and friction-free Atwood's machine mounted in a uniform time-constant gravitational field. The axle of the upper pulley is belted to an energy storing device at the upper level marked simply "output". With no leads, the dumbwaiter moves freely and without energy loss or gain. With the appropriate clock setting, Morrison's argument is as follows:

"Place an atom in the lower dumbwaiter pan, at gravitational potential ϕ_1, and an identical atom, but excited to its first quantum level above ground, in the second pan at gravitational potential ϕ_2. The upper pan is then heavy by the weight of its extra energy content, $mg = g\Delta E/c^2$. The heavier pan will fall. We can allow it to reach a small velocity, and then keep it from accelerating further by drawing an output current from the coupled generator, storing the energy in the storage cells. When the pans have exchanged places, I let the excited atom down below to decay to its ground state. In the successful trials, the photon emitted comes up to the other pan, where it strikes the upper atom, in its ground state. Were the photon to excite the upper atom to the first excited level, I should have restored the initial condition, and yet have collected energy in the storage cell. I must avoid this by the hypotesis of the first law [energy conservation]. This I can do if I realize that the photon is red shifted, having insufficient energy to excite the atom by just the amount stored in the cell."

With this reasonings, Morrison claims that the red-shift formula can be derived using only the equivalence between gravitational and inertial mass and the need to obey the overall conservation of energy.

There are, however, two major problems with these reasonings: first, they do not take into account that the energy levels of the atoms should be distorted by the gravitational field. Second, the isolated system is comprised by the atoms plus the gravitational field. Indeed, even the idea of energy in the realm of Einstein's gravitation theory is not as clear as in Newton's. Despite

this last consideration, let us analyse Morrison's argument and see if it can be reformulated.

Consider now the arrangement of figure 3 where we show the same apparatus with the atoms energy levels on its right side. The atom1 has its first excited level lowered by the gravitational field by an amount of δE with respect to atom2. Due to its greater energy content, $mg = g(\Delta E - \delta E)/c^2$, atom1 falls down freely. As atom1 arrives at the bottom, its energy content is now $mg = g\Delta E/c^2$, since the gravitational field has changed with height. The gravitational field has lost the energy amount δE realizing work on the levels of atom1. Now this configuration implies that the difference between the ground and excited levels of atom2 is $(\Delta E - \delta E)$. Let atom1 decay. We might build the Atwood's machine to have a height such that the red-shift of the emitted photon is exactly δE. The photon thus have the exactly amount of energy to excite atom2 and the process, thus, continues. The system atoms-plus-gravitational-field is isolated and energy was conserved since the amount of energy given by the field to atom1 was restored when the field extracted energy from the photon by virtue of the red-shift. If Atwood's machine is not calibrated as above, the photon will not be capable of exciting atom2 and the process will stop. In this case, we still have energy conservation when considering the system atoms-plus-gravitational-field-plus-photon (red-shifted).

The lesson to learn is that energy conservation is not due to the photon red-shift. Indeed, in Morrison's original experiment (with energy drain *and* the distortion of levels) it is possible to calibrate Atwood's machine in such a way that the photon still has sufficient energy to excite atom2. In this case, energy was taken from the gravitational field reducing its mass (in a process we might call evaporation). Since the mass responsible for the gravitational field is finite, this process of energy extraction from the gravitational field must stop (thus, in the formulae above, when energy drain is considered, we should also take into account the change δg of gravitational acceleration by virtue of mass loss). We also do not expect this process of energy extraction to have high efficience since we are supposing that we can control the time when the atom decays, something we cannot do.

In the second experiment, Morrison introduces the concept of negative mass. For this experiment we will use a more elaborated version[6] having the same physical content. The experiment is as follows:

Take a particle-antiparticle pair with equal positive and negative masses respectively and place it at rest on a gravitational field. Since the pair has no net weight it might be suspended adiabaticaly to some height L. Then, let the pair annihilate. The produced pair of photons will travel back to height zero suffering a blue-shift. Now, at the botton, let the photons produce the pair again. Since the photon energy is now greater, because of the blue-shift, the pair will have some extra kinetic energy with respect to the beginning of the process. The conclusion is that energy was created[5] violating the energy conservation principle.

The problem here is the same. The pair is considered isolated. However, the isolated system is the pair-plus-gravitational-field. In this case, the extra kinetic energy gained by the pair is exactly the amount extracted from the gravitational field with the photon's blue-shift. When considering the system pair-plus-gravitational-field, energy is conserved in exactly the same way as above. The process is just an inventive way, although not too efficient, of extracting energy from the gravitational field.

The fail of the energy-conservation argument can not be accepted.

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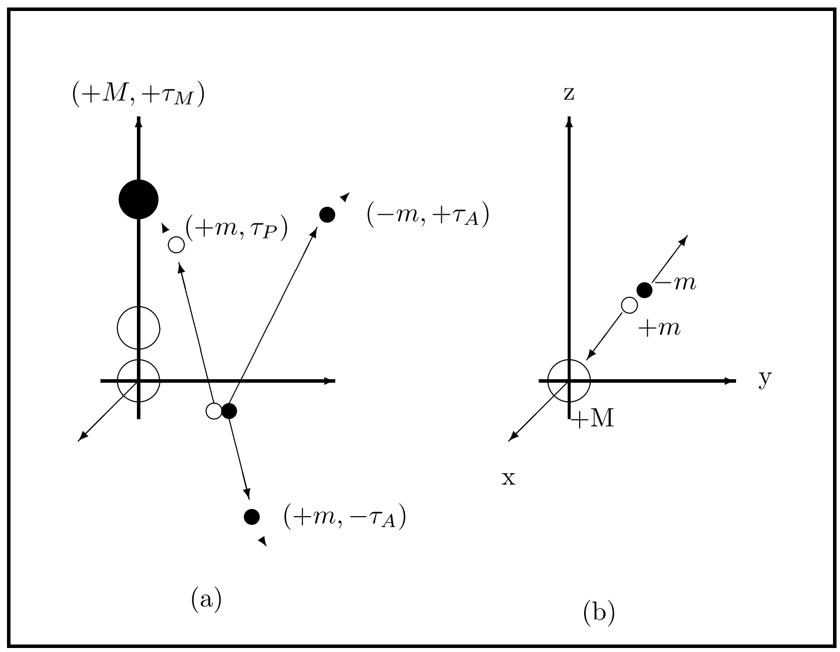

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Figure 1: Behavior of a particle-antiparticle pair in the presence of a gravitational field generated by a body of mass +M. (a) in the four-space (b) in the three-space. Particle is represented by empty circle and antiparticle by filled circle.

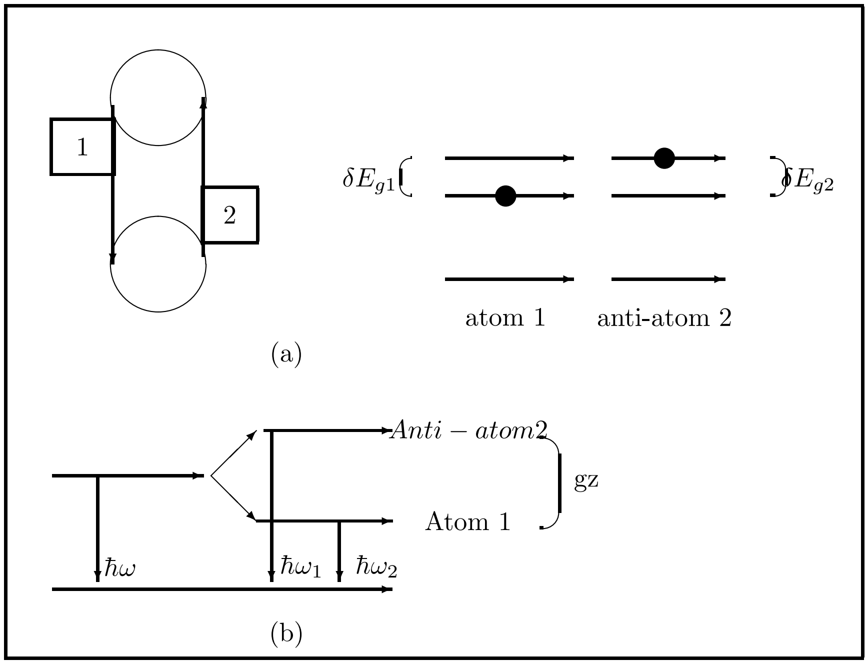

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Figure 2: The matter-antimatter experiment. (a) an Atwood machine with two pans. The higher contains matter while the lower antimatter. The atomic levels of the atom and anti-atom are also represented. (b) The level diagram of atoms and anti-atoms in the approximate gravitational potential gz.

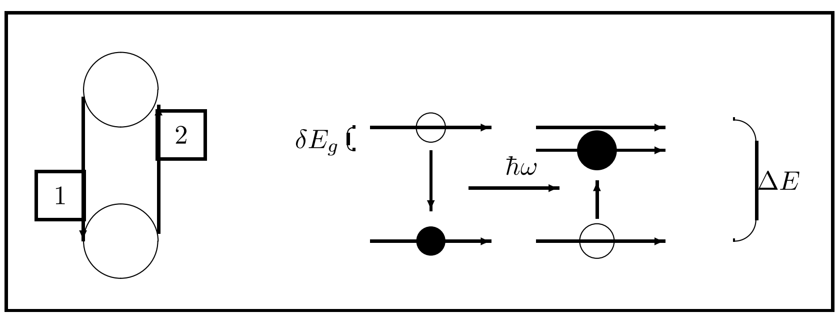

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Figure 3: The field takes from the photon the amount of energy it gave to the first atom when accelerating downward. This amount is exactly the difference of energy levels of the second atom now placed at the top of the Atwood machine.